$$\vec{u}+\vec{v}=\vec{v}+\vec{u}$$

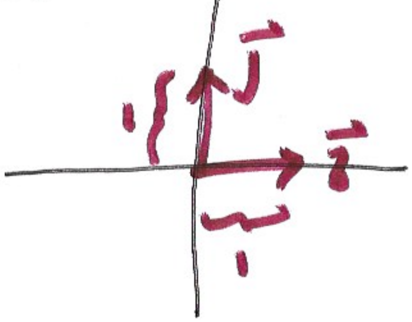
$$(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$$

$$k(c\vec{u})=(kc)\vec{u}$$

$$c(\vec{u}+\vec{v})=c\vec{u}+c\vec{v}$$

$$(k+c)\vec{u}=k\vec{u}+c\vec{u}$$

2D



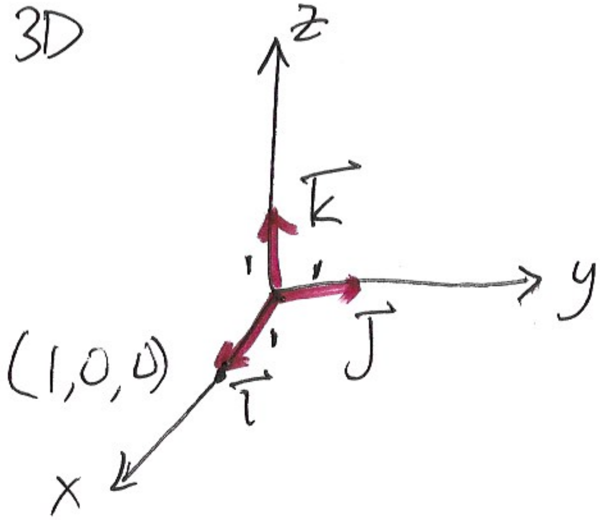

$$\|\vec{i}\|=\|\vec{j}\|=1$$

$$\vec{i}=\langle 1,0,0\rangle$$

$$\vec{j}=\langle 0,1,0\rangle$$

$$\vec{k}=\langle 0,0,1\rangle$$

3D




$$\vec{v}=\langle v_1, v_2, v_3\rangle$$
$$=\langle v_1,0,0\rangle+\langle 0,v_2,0\rangle+\langle 0,0,v_3\rangle$$
$$=v_1\langle 1,0,0\rangle+v_2\langle 0,1,0\rangle+v_3\langle 0,0,1\rangle$$
$$=v_1\vec{i}+v_2\vec{j}+v_3\vec{k}$$

LINEAR COMBINATION OF $\vec{i}, \vec{j}, \vec{k}$

$\langle 3,-5,-2\rangle = 3\vec{i}-5\vec{j}-2\vec{k}$

$4\vec{k}-2\vec{i} = \langle -2,0,4\rangle$

$\hookrightarrow -2\vec{i}+0\vec{j}+4\vec{k}$

$\|\vec{i}\| = \|\vec{j}\| = \|\vec{k}\| = 1$

$\|k\vec{u}\| = |k|\,\|\vec{u}\|$

FIND A UNIT VECTOR IN THE SAME DIRECTION AS $\vec{v} = \langle -2,1,3\rangle$

$\downarrow$

$\vec{u}$

$\vec{u} = k\vec{v} \quad (k>0)$

$\|\vec{u}\| = 1$

$\|k\vec{v}\| = 1$

$|k|\,\|\vec{v}\| = 1$

$k\|\vec{v}\| = 1$

$k = \dfrac{1}{\|\vec{v}\|}$

$$\vec{u} = \frac{1}{\|\langle -2,1,3\rangle\|}\langle -2,1,3\rangle$$

$$= \frac{1}{\sqrt{(-2)^2+1^2+3^2}}\langle -2,1,3\rangle$$

$$= \frac{1}{\sqrt{14}}\langle -2,1,3\rangle$$

$$= \frac{\sqrt{14}}{14}\langle -2,1,3\rangle$$

$$= \left\langle -\frac{\sqrt{14}}{7}, \frac{\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}\right\rangle$$

$$\boxed{\text{IF } \vec{v}\neq\vec{0},\quad \vec{u} = \frac{1}{\|\vec{v}\|}\vec{v} = \frac{\vec{v}}{\|\vec{v}\|} \text{ IS } \cancel{\text{A}}\ \text{THE UNIT VECTOR IN THE SAME DIRECTION AS } \vec{v}}$$

$\vec{v} = \langle -2, 1, 3 \rangle$

FIND ~~A~~ THE VECTOR WITH MAGNITUDE 7

IN THE OPPOSITE DIRECTION AS $\vec{v}$

$\|\vec{u}\| = 1$ $\qquad$ $\vec{u} = \left\langle -\frac{\sqrt{14}}{7}, \frac{\sqrt{14}}{14}, \frac{3\sqrt{14}}{14} \right\rangle$

SAME DIR AS $\vec{v}$

$$-7\vec{u} = -7\left\langle -\frac{\sqrt{14}}{7}, \frac{\sqrt{14}}{14}, \frac{3\sqrt{14}}{14} \right\rangle$$

$$= \left\langle \sqrt{14}, -\frac{\sqrt{14}}{2}, \frac{-3\sqrt{14}}{2} \right\rangle$$

---

DOT PRODUCT $\vec{u} \cdot \vec{v}$ (A/K/A SCALAR PRODUCT)

NOT SCALAR MULTIPLICATION $k\vec{u}$

IF $\vec{u} = \langle u_1, u_2, u_3 \rangle$

AND $\vec{v} = \langle v_1, v_2, v_3 \rangle$

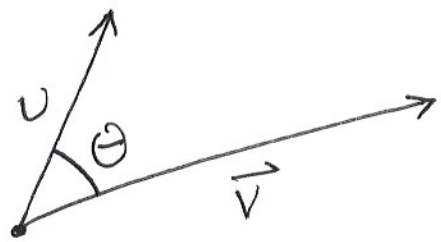
THEN $\vec{u} \cdot \vec{v} = \cancel{\langle u_1v_1, u_2v_2, u_3v_3 \rangle}$

$$\underbrace{u_1v_1 + u_2v_2 + u_3v_3}_{\text{SCALAR}}$$

ALGEBRAIC DEFINITION

$$(3\vec{k}-2\vec{j})\cdot(4\vec{i}+\vec{j}-\vec{k})$$
$$=\langle 0,-2,3\rangle\cdot\langle 4,1,-1\rangle$$
$$=0\cdot 4+(-2)\cdot 1+3\cdot(-1)$$
$$=-5 \xrightarrow{<0}$$ SO, THE ANGLE BETWEEN $3\vec{k}-2\vec{j}$ AND $4\vec{i}+\vec{j}-\vec{k}$ IS OBTUSE

GEOMETRIC DEFINITION OF $\vec{u}\cdot\vec{v}$




SAME INITIAL POINT

$$\vec{u}\cdot\vec{v}=\|\vec{u}\|\,\|\vec{v}\|\cos\theta$$

IF $\vec{u},\vec{v}\neq\vec{0}$

$\|\vec{u}\|, \|\vec{v}\|>0$

SO SIGN OF $\vec{u}\cdot\vec{v}$ DEPENDS ON $\cos\theta$

$\cos\theta>0 \longleftrightarrow \theta\in[0,\frac{\pi}{2}) \longleftrightarrow \theta$ IS ACUTE $\longleftrightarrow \vec{u}\cdot\vec{v}>0$ ($\vec{u},\vec{v}$ ARE IN THE SAME GENERAL DIR)

$\cos\theta<0 \longleftrightarrow \theta\in(\frac{\pi}{2},\pi] \longleftrightarrow \theta$ IS OBTUSE $\longleftrightarrow \vec{u}\cdot\vec{v}<0$ ($\vec{u},\vec{v}$ ARE IN GENERALLY OPPOSITE DIRECTIONS)

$\cos\theta=0 \longleftrightarrow \theta=\frac{\pi}{2} \longleftrightarrow \vec{u}\perp\vec{v} \longrightarrow \vec{u}\cdot\vec{v}=0$

(IE. $90^\circ$) (IE. $\vec{u},\vec{v}$ ARE PERPENDICULAR)

DEF'N: IF $\vec{u} \cdot \vec{v} = 0$

THEN WE SAY $\vec{u}, \vec{v}$ ARE ORTHOGONAL

(I.E. $\vec{u} \perp \vec{v}$ OR $\vec{u} = \vec{0}$ OR $\vec{v} = \vec{0}$)

eg. IF $\vec{u} = \langle 3, 1, -4 \rangle$ AND $\vec{v} = \langle -2, k, -5 \rangle$ ARE PERPENDICULAR, FIND $k$

$$\vec{u} \cdot \vec{v} = 0$$

$$3(-2) + 1(k) + (-4)(-5) = 0$$

$$-6 + k + 20 = 0$$

$$k = -14$$

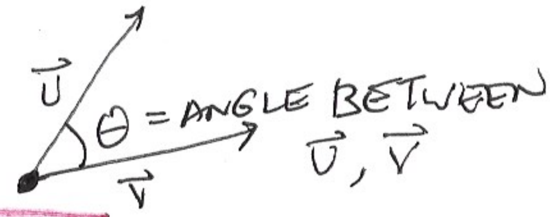
CRITICAL CONCEPTS

① IF $\vec{u} \parallel \vec{v}$, THEN $\vec{u} = k\vec{v}$ (OR $\vec{v} = k\vec{u}$)

② IF $\vec{u} \perp \vec{v}$, THEN $\vec{u} \cdot \vec{v} = 0$

FIND THE ANGLE BETWEEN

$\vec{u} = \langle 2, 0, -1 \rangle$ AND $\vec{v} = \langle -1, 1, 3 \rangle$




$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos\theta$$

$$\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} = \cos\theta \longrightarrow \boxed{\theta = \cos^{-1} \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}}$$

ANGLE BETWEEN $\vec{u}, \vec{v}$

$$\theta = \cos^{-1} \frac{\langle 2, 0, -1 \rangle \cdot \langle -1, 1, 3 \rangle}{\|\langle 2, 0, -1 \rangle\| \, \|\langle -1, 1, 3 \rangle\|}$$

$$= \cos^{-1} \frac{-2 + 0 + 3}{\sqrt{5} \sqrt{11}}$$

$$= \cos^{-1} \left( \frac{-5}{\sqrt{55}} \right)$$

$$= \cos^{-1} \left( -\frac{\sqrt{55}}{11} \right)$$